Numerical Methods for Hyperbolic Conservation Laws

Module-No./Abbreviation	Credits	Workload	Term	Frequency	Duration
CE-WP17/NMfHCL	6 CP	180 h	3rd Sem.	Winter term	1 Semester
Courses			**Contact hours**	**Self-Study**	**Group Size:**
Numerical Methods for Hyperbolic Conservation Laws			4 SWS (60 h)	120 h	No Restrictions

Prerequisites

Basic knowledge about: ordinary differential equations, numerical integration, and numerical methods for the solution of large linear and non-linear systems of equations

Learning goals / Competences

Students should attain familiarity with numerical methods for the solution of differential equations, in particular hyperbolic conservation laws. This includes understanding the notion of entropy solutions and being able to construct stable numerical schemes that are capable of finding such solutions.

After successfully completing the module, the students shall be able to

- design, implement and use numerical methods for computer solution of scientific problems involving differential equations,
- understand properties of different classes of differential equations and their impact on solutions and proper numerical methods,
- understand the different concepts of solutions to hyperbolic conservation laws and their physical interpretations, know how to select appropriate numerical methods that capture the physically correct solutions,
- use software for solving differential equations with understanding of fundamental methods, properties, and limitations.

Content

- 1st week:
  Introduction to PDE's; classification of PDE's; well-posedness; outline of the course
- 2nd – 3th week:
  Heat equation Setting; well-posedness; space discretization; properties of the discretization; finite volumesin1D and 2D; stability of ODEs (repetition); time discretization
- 4th – 5th week:
  First order hyperbolic equations and characteristics; example: Burgers equation; crash of characteristics; discontinuous solutions; basic discretizations; characteristics for linear advective systems; linear Riemann problems
- 6th week:
  Basic discretizations finite volume methods; linearization of nonlinear conservation laws; boundary conditions
- 7th –8th week:
  Convergence theory for linear methods notation; convergence, consistency and stability; verifying stability: CFL numbers; Von Neumann analysis
- 9th –10th week:
  Weak solutions and viscosity solutions weak solutions; viscosity limits and modified equations; Lax entropy condition; applications of entropy conditions; explicit entropy solutions to Riemann problems; weak entropy conditions; entropy pairs

• 11^{th} – 12^{th} week: Monotone schemes, Consistent methods; idea of monotone schemes; properties of monotone schemes; the Godunov scheme • 13^{th} week: Higher Order Finite volume methods for non-linear hyperbolic equations Lax-Wendroff scheme; TVD schemes, slope/flux limiters • 14^{th} week: Summary /exam preparation
Teaching methods / Language Lecture (3h / week), Exercises (1h / week) / Homework (30) / English
Mode of assessment Written examination (120 min, 100%) / Homework
Requirement for the award of credit points Passed homework and passed final module examination
Module applicability MSc. Computational Engineering
Weight of the mark for the final score 6 %
Module coordinator and lecturer(s) Prof. Dr. P. Henning, Assistants
Further information